AB review Test

1)

$\int_0^{\infty} x^2 e^{-x^3}\, dx$ is

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

(Think – just put in infinity as a "number" then think limits…).

$u = -x^3$

$du = -3x^2 dx$

$\frac{du}{-3} = x^2 dx$

$\int e^u \frac{du}{-3} = -\frac{1}{3}e^u = -\frac{1}{3}e^{-x^3}\Big|_0^{\infty} = -\frac{1}{3e^{x^3}}\Big|_0^{\infty}$

So… $\left[-\frac{1}{3e^{\infty}}\right] - \left[-\frac{1}{3e^0}\right] = +\frac{1}{3}$

2)

$\int_1^e \left(\frac{x^2-1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

$\int_1^e \left[\frac{x^2}{x} - \frac{1}{x}\right] dx = \int_1^e \left[x - \frac{1}{x}\right] dx = \frac{x^2}{2} - \ln|x|\Big|_1^e =$

$= \left[\frac{e^2}{2} - \ln e^1\right] - \left[\frac{1^2}{2} - \ln|1|\right] = \frac{e^2}{2} - 1 - \frac{1}{2}$

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3)

$$\int \frac{1}{x^2-6x+8}\,dx =$$

Don't Do This (requires "partial fractions")

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right|+C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right|+C$

(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right|+C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right|+C$

(E) $\ln\left|(x-2)(x-4)\right|+C$

4) CALCULATOR PROBLEM

Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

$f'(x) = 6e^{2x}$ $g'(x) = 18x^2$

$\parallel$ means same slope

$6e^{2x} = 18x^2$
$e^{2x} = 3x^2$ put in Calc to find intersection of these

AB review Test

5)

If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$ $\cos(e^{-x}) \cdot (-e^{-x})$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

6)

If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

2nd !! fund ..

$$\frac{d}{dx}\int_0^x \sqrt{t^3+1}\,dt = \sqrt{x^3+1} \dashrightarrow \sqrt{2^3+1} = 3$$

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7) repeat

$$\int_1^e \left(\frac{x^2-1}{x}\right)dx =$$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

8)

Which of the following are antiderivatives of $f(x)=\sin x\cos x$?

✓ I. $F(x)=\frac{\sin^2 x}{2}$

II. $F(x)=\frac{\cos^2 x}{2}$ ← neg. would work

✓ III. $F(x)=\frac{-\cos(2x)}{4}$

$u = \sin x$

$du = \cos x\,dx$

so $\int u\,du = \frac{u^2}{2}+C$ or

I. $\boxed{\frac{\sin^2 x}{2}+C}$

$\rightarrow + \frac{\sin 2x}{4}\cdot\frac{2}{1} = \frac{1}{2}\sin 2x \rightarrow 2\sin x\cos x$

so: $\sin x\cos x$ yes ✓

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

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9)

If $f(x)=\dfrac{e^{2x}}{2x}$, then $f'(x)=$

$$\frac{2x(2e^{2x})-e^{2x}\cdot 2}{4x^2}$$

now GCF

$$\frac{\cancel{2}e^{2x}[2x-1]}{2\cancel{4}x^2}$$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

10)

If $f(x)=\ln\left|x^2-1\right|$, then $f'(x)=$ $\frac{1}{u}du$ or $\frac{1}{x^2-1}\cdot 2x$

no ABS!!

(ABS for integrating only)

(A) $\left|\dfrac{2x}{x^2-1}\right|$

(B) $\dfrac{2x}{\left|x^2-1\right|}$

(C) $\dfrac{2|x|}{x^2-1}$

(D) $\dfrac{2x}{x^2-1}$

(E) $\dfrac{1}{x^2-1}$

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11)

$\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-\frac{t}{2}}+C$ (B) $e^{-\frac{t}{2}}+C$ (C) $e^{\frac{t}{2}}+C$ (D) $2e^{\frac{t}{2}}+C$ (E) $e^{t}+C$

$u = \frac{t}{2}$

$du = \frac{1}{2}dt$

$\frac{du}{1/2} = dt$

$\left(\frac{du}{1/2} = 2du\right)$

$\frac{1}{2}\int e^{u}\cdot 2\,du$

$= \int e^{u}du$

$= e^{u}+C = e^{t/2}+C$

12)

$\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\frac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

$u = x^3+1$

$du = 3x^2dx$

$\int \frac{1}{\sqrt{u}}du = \int u^{-1/2}du$

$= \frac{u^{1/2}}{1/2}+C$

$= 2\sqrt{x^3+1}+C$